Technical Comments

Pressure Ratio Correction Factor when Utilizing the Hydraulic Analogy

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Nomenclature

a	=	sonic velocity
A/A^*	=	one-dimensional area ratio
C_p	=	static pressure correction factor, defined by Eq. (4)
Fr	=	Froude number $u/(gh)^{1/2}$
g	=	acceleration of gravity
h	=	height of water above table
M	=	Mach number (u/a)
P	=	fluid static pressure
u	=	local velocity
γ	=	specific heat ratio

Subscripts

0	=	stagnation condition
γ	=	actual gas condition
$\gamma = 2$	=	water table condition [Eq. (1), etc.]

THE hydraulic analogy relates the frictionless two-dimensional flow of a liquid with a free surface to the flow of a compressible gas. A water table is an experimental tool utilizing the hydraulic analogy, in which water is flowed on a horizontal glass plate, around or through the model to be tested. For many years, the water table has been used to aid in visualization of the complex flow patterns that can exist when a fluid encounters a solid boundary.

Adams[1] presents an excellent set of correction factors which can be used to apply the results from the water table to two-dimensional axisymmetric gas systems. In his correction factors for pressure, density, and temperature, he uses the Mach number for the gas in both the numerator and denominator and calculates this Mach number from an additional correction factor on the water table Froude number and the water's analogous specific heat ratio of 2.0. This method is an improvement over that of Byrd and Williams[2] where the Mach number used in the pressure correction factor was that of the water. However, this author feels a still more accurate method of calculating this correction factor can be made.

The fundamental relation between the pressure of the hypothetical gas of $\gamma = 2.0$ and the height of the water is

$$(P/P_0)_{\gamma=2} = (h/h_0)^2 \tag{1}$$

To calculate the pressure ratio of a gas of any specific heat ratio, it is necessary to define a correction factor C_p where

$$(P/P_0)_\gamma = C_p(P/P_0)_{\gamma=2.0} \tag{2}$$

It is possible to calculate C_p by using the well-known isentropic relation

$$P/P_0 = \{1 + [(\gamma - 1)/2]M^2\}^{\gamma/1-\gamma} \tag{3}$$

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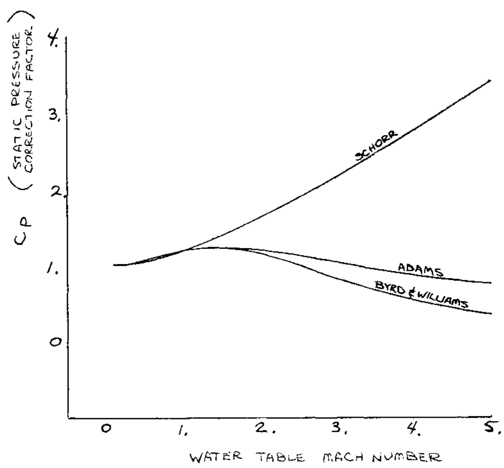

Fig. 1 Comparison of various definitions of C_p.

Note that the left side of Eq. (2) is for the gas of interest, whereas the right side is for the hypothetical water (gas) of $\gamma = 2.0$. Now, placing (3) into (2) and solving for C_p,

$$C_p = \frac{\{1 + [(\gamma - 1)/2]M_\gamma^2\}^{\gamma/1-\gamma}}{[1 + (M_{\text{water}}^2/2)]^{-2}} \tag{4}$$

The three different definitions of C_p have been used for a γ of 1.4 (air) and are presented in Fig. 1.

It should be pointed out that this argument is largely academic, as the practical way to gather data from the water table is to calculate the Mach number of the water from the height measurement and the equation

$$M_{\gamma=2} = Fr_{\gamma=2} = \{2[(h_0 - h)/h]\}^{1/2} \tag{5}$$

then calculate the area ratio from the one-dimensional isentropic area ratio equation

$$\frac{A}{A^*} = \frac{1}{M_{\gamma=2}}\left[\left(\frac{2}{\gamma+1}\right) \times \left(1 + \frac{\gamma-1}{2}M^2_{\gamma=2}\right)\right]^{(\gamma+1)/2(\gamma-1)} \tag{6}$$

where $\gamma = 2.0$

Since, as Adams mentioned, A/A^* is the same for the gas and analogous water table system, Eq. (6) can be reused with the gas specific heat ratio to solve for M_γ. Then, finally, the static pressure ratio P/P_0 can be obtained by use of Eq. (3).

References

[1] Adams, D. M., "Application of the Hydraulic Analogy to Axisymmetric Nonideal Compressible Gas Systems," *Journal of Spacecraft and Rockets*, Vol. 4, No. 3, March 1967, pp. 359–363.

[2] Byrd, J. L. and Williams, J. G., "Static Pressure Distribution Along an Inclined, Setback Plate with Attached Jet Using the Hydraulic Analogy," Army Missile Command Rept. RG-TR-63-15, Aug. 1963, Redstone Arsenal, Ala.